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Further Results on the Impact of Quality of Wireless Sensor Links on Decentralized Detection Performance

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Abstract: In this paper we consider the impact of quality of wireless sensor links on the overall detection performance of a large sensor network system. Independent and identical sensors gather observations regarding the presence or the absence of a phenomenon of interest and then transmit their binary decisions to a fusion center over parallel, non-interfering but slow Rayleigh fading, wireless links. We derive asymptotic error exponents of the probability of false alarm and the probability of miss at the fusion center for the following cases: (a) BPSK modulation and (i) maximal ratio combining (ii) equal gain combining (iii) decision fusion (b) BFSK modulation and (i) square law combining (ii) decision fusion. In the case of BPSK, the EGC performs the best for low and moderate SNR, with the DF achieving the next best performance. The DF scheme performs the best for large SNR values, whereas the MRC performs the best for very low SNR values. Similar relative performance results were obtained by others for the case of a finite number of sensors. In the case of BFSK, square law combining outperforms DF, except for large SNR values. Finally, we show how the false alarm and the detection probabilities of the decision of a sensor, as seen at the fusion center, are altered by changes to the threshold of the matched filter receiver.

I. INTRODUCTION

Performances of decentralized detection (DD) systems employing a set of geographically separated sensors have been investigated for the past couple of decades. In the earlier studies, the transmission links from the distributed sensors to a fusion center (FC) were assumed to be error free. However, because of recent interest in wireless sensor networks, many authors have analyzed the performance of these DD systems in which the transmissions from the sensors to the FC are subject to channel fading and noise [1-6]. Apart from bandwidth and power requirements, the performance of a wireless sensor DD system also depends on many other factors such as the decision fusion rules, channel error control coding, sensor quality etc. For the case of a finite number of sensors, [4] examines the variations in the false alarm and the detection probabilities of a DD system due to errors caused by the sensor links. For a guaranteed minimum sensor-to-fusion center link average SNR, a counting rule at the fusion center, and different binary modulation schemes, it points out the necessity of having sensors with a specific minimum quality in order to achieve an asymptotically (as the number of sensors tends to infinity) vanishing probability of error. With similar assumptions, the asymptotic error exponents of decision counting rule were derived in [5] using large deviation theory.

Fusion of binary decisions transmitted over fading channels has particularly important applications in low-cost low-power wireless sensor networks. In [6], the authors have formulated the parallel fusion problem with a fading channel layer and derived the optimal likelihood ratio (LR) based fusion rule with binary local decisions along with three other sub-optimal fusion rules: a two-stage approach using the Chair-Varshney fusion rule, a maximal ratio combining (MRC) fusion statistic, and an equal gain combiner (EGC) fusion statistic. Performance analysis of the optimal and the sub-optimal rules was carried out for the case of a finite number of sensors.

In this paper, we examine further the impact of quality of wireless sensor links on the performance of a DD system. Specifically, we address the following issues: (a) for a large sensor network, the asymptotic error exponents of maximal ratio combiner (MRC) and equal gain combiner (EGC) at the fusion center for binary PSK modulation and the error exponents of square law combiner (SLC) for binary FSK modulation; comparison to the error exponents obtained with the decision fusion rule, (b) for binary PSK and FSK modulations, the effect of matched filter threshold on the detection performance. The last issue requires a bit of elaboration. For equally likely binary data, it is well known that the optimal (in the sense of minimum probability of error) matched filter threshold for receiving PSK in AWGN is zero (this optimality holds for slow Rayleigh fading channel also). However, in a wireless sensor network, which is deployed to detect the presence or the absence of a phenomenon of interest (POI), the Neyman-Pearson criterion is of interest. In such a situation, the zero-threshold for the matched filter output need not be optimal. Similar situation arises for the FSK modulation. For a single sensor-to-fusion link, the probability of detection (β), as a function of probability of false alarm (α) at the fusion center, is evaluated. Throughout the paper it is assumed that all the sensors are identical and that the decisions made by them, conditioned on a hypothesis, are all statistically independent. It is also assumed that non-interfering parallel links exist for connecting sensors to the fusion center.

II. ASYMPTOTIC PERFORMANCE OF MRC, EGC, DECISION, AND SQUARE LAW COMBINING

In the context of sensor networks, where all the sensors need not make identical decisions on the hypothesis of

interest, it was pointed out in [6] that the MRC of received signals from different sensors does not provide the best detection performance. Here, we evaluate the rate at which the asymptotic error goes to zero for MRC, EGC, and SLC and compare their rates with that of the decision counting rule. Let $P_f > 0$ and $P_d < 1$ denote the probability of false alarm and the probability of detection, respectively, of a sensor.

a) MRC

In a WSN of n sensors, consider the situation that k out of n sensors decide '1' (presence of POI) and that the remaining $n-k$ sensors decide otherwise. Without any loss of generality, it can be assumed that the first set of k sensors had decided binary '1'. If the sensors use binary PSK signaling to transmit their data, then upon matched filtering, the maximal ratio combiner output for the Rayleigh faded PSK signals received in zero mean AWGN is given by

$$S = \sum_{j=1}^k h_j^2 - \sum_{j=k+1}^n h_j^2 + \sum_{j=1}^n h_j n_j \quad (1)$$

Where $h_j > 0$ is the channel gain of the j^{th} link. Implementation of MRC requires the knowledge of the channel states, $\{h_j\}$. h_j , $j = 1, 2, \dots$ are all *iid* as exponential with mean σ_R^2 and n_j , $j = 1, 2, \dots$ are *iid* zero mean Gaussian noise with variance σ^2 , which are independent of h_j . However, for very large n and under the hypothesis of no POI (H_0), S can be treated as the sum of n *iid* samples of the form shown below:

$$S = \sum_{j=1}^n (y_j + h_j n_j) \quad (2)$$

$$\text{where } y_j = \begin{cases} h_j^2 & \text{with probability } P_f \\ -h_j^2 & \text{with probability } (1 - P_f) \end{cases}$$

Using large deviations, we can find the rate with which the false alarm error probability at the fusion center approaches zero [7]:

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{F0} &= \lim_{n \rightarrow \infty} P \left(\sum_{i=1}^n Z_i \geq 0 \mid H_0 \right) \\ &= e^{-nD_{\text{FMRC}}} + \text{terms going to zero faster than the first} \end{aligned} \quad (3)$$

where Z_i are *i.i.d* variables specified by

$$Z_i = I_i h_i^2 - (1 - I_i) h_i^2 + n_i h_i - C, \quad (4)$$

I_i is the indicator function specifying the decision of the sensor i , viz., $I_i = 1$, when the sensor decides H_1 , and $I_i = 0$, when it decides H_0 , and C is a constant threshold value. In order to ensure that the false alarm error goes to zero in the limit as n goes to infinity, it is required that the constant C be chosen to yield a negative expected value of Z_i . Hence,

$$C^* = \frac{C}{\sigma_R^2} > (2 P_f - 1) \quad (5)$$

From [7], the error exponent in (3) is given by

$$D_{\text{FMRC}} = -\log(\rho_{0\text{MRC}}), \quad (6)$$

$$\rho_{0\text{MRC}} = \inf_{t > 0} \phi_Z(t), \quad (7)$$

and $\phi_Z(t)$ denotes the moment generating function of the variable Z_i . Some routine evaluations yield

$$\phi_Z(\tau) = e^{-\tau C^*} \left[\frac{(1 - P_f)}{1 - a \tau^2 + \tau} + \frac{P_f}{1 - a \tau^2 - \tau} \right], \quad (8)$$

where $\tau = t \sigma_R^2$, $\theta = \frac{\sigma_R^2}{\sigma^2}$, $a = \frac{1}{2\theta}$, and θ can be defined as

the average signal-to-noise ratio (SNR) of the Rayleigh channel. It can be shown that the required infimum in (7) occurs over the interval, $\frac{-1 + \sqrt{1 + 4a}}{2a} > \tau > 0$. Similarly, the asymptotic probability of a miss is given by

$$\begin{aligned} \lim_{n \rightarrow \infty} P_{M0} &= \lim_{n \rightarrow \infty} P \left(\sum_{i=1}^n Z_{1i} \geq 0 \mid H_1 \right) \\ &= e^{-nD_{\text{MMRC}}} + \text{terms going to zero faster than the first} \end{aligned} \quad (9)$$

where Z_{1i} in (9) is defined to be the negative of Z_i in (4). Proceeding exactly as in the case of false alarm probability, we obtain the following:

$$C^* < (2 P_d - 1) \quad (10)$$

$$\phi_{Z1}(\tau) = e^{\tau C^*} \left[\frac{(1 - P_d)}{1 - a \tau^2 - \tau} + \frac{P_d}{1 - a \tau^2 + \tau} \right] \quad (11)$$

$$D_{\text{MMRC}} = -\log(\rho_{1\text{MRC}}) \quad (12)$$

$$\rho_{1\text{MRC}} = \inf_{\frac{-1 + \sqrt{1 + 4a}}{2a} > \tau > 0} \phi_{Z1}(\tau) \quad (13)$$

Since we require that both the errors approach zero asymptotically, the threshold C has to satisfy both the constraints, (5) and (10).

b) EGC

For equal gain combining, the equation for S_{EGC} , that is an analog of (1) for the MRC, is given by

$$S_{\text{EGC}} = \sum_{j=1}^k h_j - \sum_{j=k+1}^n h_j + \sum_{j=1}^n n_j \quad (14)$$

Analogous to (2), under H_0 , let S_{EGC} denote the variable at the output of EGC:

$$S_{\text{EGC}} = \sum_{j=1}^n (y_j + n_j) \quad (15)$$

$$\text{where } y_j = \begin{cases} h_j & \text{with probability } P_f \\ -h_j & \text{with probability } (1 - P_f) \end{cases}$$

Proceeding exactly as done for MRC, we can obtain the asymptotic rates at which both the errors go to zero ($D_{\text{FEGC}} = -\log(\rho_{0\text{EGC}})$, $D_{\text{MEGC}} = -\log(\rho_{1\text{EGC}})$).

$$\rho_{0\text{EGC}} = \inf_{\tau > 0} \exp \left(-C_{\text{EGC}}^* \tau + \left(\tau^2 / 2\theta \right) \right) \left(1 + \frac{\sqrt{\pi} \tau}{2} e^{\tau^2 / 4} (2 P_f - 1 + \text{erf}(\tau / 2)) \right) \quad (16)$$

$$\rho_{1\text{EGC}} = \inf_{\tau > 0} \exp \left(C_{\text{EGC}}^* \tau + \left(\tau^2 / 2\theta \right) \right) \left(1 + \frac{\sqrt{\pi} \tau}{2} e^{\tau^2 / 4} (1 - 2 P_d + \text{erf}(\tau / 2)) \right) \quad (17)$$

$$\frac{2P_d-1}{2}\sqrt{\pi} > C_{\text{EGC}}^* = \frac{C}{\sigma_R} > \frac{2P_f-1}{2}\sqrt{\pi} \quad (18)$$

where the error function is given by $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2) dt$.

c) DECISION FUSION

Denoting the decision on a bit made at the fusion center, corresponding to the decision sent from sensor i as I_i , a simple decision fusion strategy is based on the counting rule[5]:

Decide H_1 iff

$$Z_n = \sum_{i=1}^n I_i \geq n \alpha_u, \quad (19)$$

where $1 > \alpha_u > 0$ is a suitable threshold that controls the fusion false alarm probability. Assuming a slow Rayleigh fading sensor-to-fusion link, the probability of false alarm α , and the probability of detection β , corresponding to an individual sensor decision made at the fusion center, are related to the sensor false alarm probability P_f , and the sensor detection probability P_d , respectively, in the following manner [4], [5]:

$$\text{PSK:} \quad \alpha = \frac{1}{2} \left(1 - \frac{1-2P_f}{\sqrt{1+\theta}} \right) \quad (20)$$

$$\beta = \frac{1}{2} \left(1 + \frac{1-2P_d}{\sqrt{1+\theta}} \right)$$

$$\text{Noncoherent FSK:} \quad \alpha = P_f + \frac{1-2P_f}{2+\theta} \quad (21)$$

$$\beta = P_d + \frac{1-2P_d}{2+\theta}$$

Using large deviations we obtain the following, when $\alpha < \alpha_u < \beta$ [5], [7]:

$$\lim_{n \rightarrow \infty} P_{F0} = P\left(\frac{Z_n}{n} \geq \alpha_u \mid H_0\right) = e^{-nD_{\text{FDF}}} \quad (22)$$

$$D_{\text{FDF}} = -\log \left[\left(\frac{\alpha}{\alpha_u} \right)^{\alpha_u} \left(\frac{1-\alpha}{1-\alpha_u} \right)^{1-\alpha_u} \right] \quad (23)$$

Similarly,

$$\lim_{n \rightarrow \infty} P_{M0} = P\left(\frac{Z_n}{n} < \alpha_u \mid H_1\right) = e^{-nD_{\text{MDF}}} \quad (24)$$

$$D_{\text{MDF}} = -\log \left[\left(\frac{\beta}{\alpha_u} \right)^{\alpha_u} \left(\frac{1-\beta}{1-\alpha_u} \right)^{1-\alpha_u} \right] \quad (25)$$

d) SQUARE LAW COMBINING OF FSK SIGNALS

Consider the case where the sensors use binary FSK signaling to transmit their data. Let f_{0k}, f_{1k} be the frequencies by which the sensor k sends binary bits, $I_k=0, I_k=1$, respectively. After square law combining of the n branch signals, let S_0, S_1 denote the square law outputs that detect the

frequencies, f_{0*}, f_{1*} , respectively. Under H_0, S_0 and S_1 can be represented by the following equations:

$$S_0 = \sum_{k=1}^n y_{0k} = \sum_{k=1}^n (1-I_k)X_{1k} + I_k X_{0k} \quad (26)$$

$$S_1 = \sum_{k=1}^n y_{1k} = \sum_{k=1}^n I_k X_{1k} + (1-I_k) X_{0k}$$

Where X_{1k} is distributed as exponential with mean $(\sigma_R^2 + \sigma^2)$ and X_{0k} is distributed as exponential with mean σ^2 , when the frequency f_{1k} was transmitted by the sensor k , and the distributions are interchanged when the frequency f_{0k} was sent.

Clearly,

$$S_1 - S_0 = \sum_{k=1}^n (2I_k - 1)(X_{1k} - X_{0k}) \quad (27)$$

I_k, X_{1k}, X_{0k} are all mutually statistically independent. Moreover, they are mutually independent across the index k . We are interested in the following asymptotic errors:

$$P_{F0} = \lim_{n \rightarrow \infty} P((S_1 - S_0) \geq nC \mid H_0) \quad (28)$$

$$P_{M0} = \lim_{n \rightarrow \infty} P((-S_1 + S_0) \geq nC \mid H_1) \quad (29)$$

Proceeding exactly as in III (a), we get the following asymptotic error exponents:

$$D_{\text{FSLC}} = -\log \left(\inf_{1/(1+\theta) < \tau < 1} \exp(-\tau C_{\text{SLC}}^* \left(\frac{P_f}{(1-\tau(1+\theta))(1+\tau)} + \frac{1-P_f}{(1+\tau(1+\theta))(1-\tau)} \right)) \right) \quad (30)$$

$$D_{\text{MSLC}} = -\log \left(\inf_{1/(1+\theta) < \tau < 1} \exp(\tau C_{\text{SLC}}^* \left(\frac{P_d}{(1+\tau(1+\theta))(1-\tau)} + \frac{1-P_d}{(1-\tau(1+\theta))(1+\tau)} \right)) \right) \quad (31)$$

where

$$(2P_f-1)\theta < C_{\text{SLC}}^* = \frac{C}{\sigma_R^2} < (2P_d-1)\theta \quad (32)$$

For a fixed average channel SNR and for varying C we observe how the asymptotic rates vary for different combining schemes. All the error exponents were computed using simple MATLAB[®] programs. Figures 1 through 4 show the variations of the miss probability error exponent against the false alarm probability error exponent. From these figures and from others which were obtained, but not shown because of space limitations, we observe the following. For binary PSK, except for very low SNR, the decision fusion outperforms MRC, in the sense that for a given false alarm error exponent, the DF provides higher miss error exponent than MRC. In general, equal gain combining does better than DF and MRC. When SNR is small, performance of MRC gets better (see SNR = 5dB and 0 dB graphs, (Figs. 1-3)). We verified that under very low SNR, MRC becomes the best among the others considered. This is consistent with the fact the optimal likelihood ratio test is approximated by MRC, as indicated in [6]. At a high SNR of 10 dB (see Fig. 4, as an example), DF becomes even better, surpassing the performance of EGC under majority of the situations analyzed. Only for $(P_f = 0.001, P_d = 0.7)$, the EGC becomes better than DF at high false alarm error exponents. Individual decisions required in DF are based on coherent detection of PSK signals and hence requires the tracking of carrier phases of individual sensor-to-

fusion links. In addition, the MRC requires the channel state information, viz., channel coefficients $\{h_{ij}\}$. Hence, considering both the complexity of implementation and performance, EGC is the best choice for low to moderate SNR, whereas the DF is the best choice for large SNR values. Similar relative performance results hold true for the case of a small number of sensors [6]. For perfect sensors, viz., $P_f = 0$, $P_d = 1$, MRC is the optimal combiner. For BFSK, square law combining outperforms DF for SNR values of 0dB and 5 dB. Only for a moderate SNR of 10 dB, DF outperforms SLC slightly. In general, the best error exponents achieved with FSK are below those achieved with PSK. Considering that noncoherent FSK does not require carrier phase tracking, when FSK is chosen as the modulation scheme, SLC with FSK is a good choice for low to moderate SNR. At high SNR, DF is preferred over SLC.

III. EFFECT OF THRESHOLD ON α AND β -SINGLE SENSOR-FUSION LINK

In this section, we consider the effect of matched filter threshold on the quality of the decision made at a sensor-fusion link. Previous works on distributed detection have assumed that the matched filter threshold for making decisions on binary PSK signals was set at zero. While this is the optimum threshold for equally likely hypotheses and minimum error criterion, this need not be the only option for the DD problem. In fact, by changing the threshold, both the false alarm and the detection probabilities of a sensor decision, as seen by the fusion center, will be altered. We could then pose the question as to which modulation scheme would be better for transmitting a sensor decision to the fusion center.

(a) Binary PSK

For coherent binary PSK signals in AWGN, the probability of false alarm, conditioned on a signal level s and a threshold t , is given by

$$P(\text{false alarm} | s) = P_f \cdot P(X > t | 1 \text{ sent}) + (1 - P_f) \cdot P(X > t | 0 \text{ sent}) \quad (33)$$

where X is the matched filter output, which is Gaussian distributed with mean s and variance σ^2 . Hence,

$$P(\text{false alarm} | s) = P_f Q\left(\frac{t-s}{\sigma}\right) + (1 - P_f) Q\left(\frac{t+s}{\sigma}\right) \quad (34)$$

where $Q(\cdot)$ is one minus the CDF of a standard Gaussian variable. For a slow Rayleigh fading channel, the unconditional false alarm probability can be obtained as

$$\alpha = P_f \int_0^\infty \frac{1}{\theta} Q\left(\tau - \sqrt{y}\right) e^{-y/\theta} dy + (1 - P_f) \int_0^\infty \frac{1}{\theta} Q\left(\tau + \sqrt{y}\right) e^{-y/\theta} dy \quad (35)$$

where $\theta = \frac{E(s^2)}{\sigma^2}$ is defined as the average channel SNR and τ is set equal to t/σ .

Similarly, the probability of detection is obtained as

$$\beta = P_d \int_0^\infty \frac{1}{\theta} Q\left(\tau - \sqrt{y}\right) e^{-y/\theta} dy + (1 - P_d) \int_0^\infty \frac{1}{\theta} Q\left(\tau + \sqrt{y}\right) e^{-y/\theta} dy \quad (36)$$

(b) Binary FSK

For binary FSK signals in slow Rayleigh fading channels, let f_0 and f_1 be the two frequencies that are used for transmitting the binary decision '0' and '1', respectively. Let r_0 and r_1 be the received signal envelopes at the outputs of the noncoherent detector, corresponding to the two respective frequencies. We first show that the difference of the squared envelopes is a sufficient statistic for this sensor decision reception problem. A likelihood ratio test was derived in [8] for the case of distributed detection in diversity channels. However, in that case, only a single transmitter sends binary data to a receiver, whereas in our problem, with nonzero probability, a group of sensors send binary '1' and another group sends binary '0'.

(i) Derivation of Sufficient Statistic

Let u denote the envelope of the received narrowband tone at frequency f_0 (or f_1) and let σ^2 denote the variance of the in-phase (and the quadrature-phase) narrowband Gaussian process [9]. The density of the received envelope, when a tone is present along with noise at the input of a noncoherent filter, as well as the density of the received envelope, when noise only is present at the noncoherent filter input, can be obtained from standard textbooks [9]-[10]. Therefore, the conditional likelihood functions are given by

$$p(r_1, r_0 | u, H_0) = P_f \left[\frac{r_1}{\sigma^2} \exp\left(-\frac{r_1^2 + u^2}{2\sigma^2}\right) I_0\left(\frac{r_1 u}{\sigma^2}\right) \frac{r_0}{\sigma^2} \exp\left(-\frac{r_0^2}{2\sigma^2}\right) \right] \\ + (1 - P_f) \left[\frac{r_0}{\sigma^2} \exp\left(-\frac{r_0^2 + u^2}{2\sigma^2}\right) I_0\left(\frac{r_0 u}{\sigma^2}\right) \frac{r_1}{\sigma^2} \exp\left(-\frac{r_1^2}{2\sigma^2}\right) \right] \quad (37)$$

$$p(r_1, r_0 | u, H_1) = P_d \left[\frac{r_1}{\sigma^2} \exp\left(-\frac{r_1^2 + u^2}{2\sigma^2}\right) I_0\left(\frac{r_1 u}{\sigma^2}\right) \frac{r_0}{\sigma^2} \exp\left(-\frac{r_0^2}{2\sigma^2}\right) \right] \\ + (1 - P_d) \left[\frac{r_0}{\sigma^2} \exp\left(-\frac{r_0^2 + u^2}{2\sigma^2}\right) I_0\left(\frac{r_0 u}{\sigma^2}\right) \frac{r_1}{\sigma^2} \exp\left(-\frac{r_1^2}{2\sigma^2}\right) \right] \quad (38)$$

Let $X = \frac{u^2}{\sigma^2}$. Then, for a Rayleigh channel, X is distributed as

an exponential random variable with mean $\theta = E(u^2)/\sigma^2$. The ratio of averaged likelihoods of (37) and (38), averaged with respect to the distribution of X , leads to the following likelihood ratio test: Decide hypothesis H_1 if

$$\frac{P_d \exp\left(\frac{r_1^2}{\sigma^2(2 + 4/\theta)}\right) + (1 - P_d) \exp\left(\frac{r_0^2}{\sigma^2(2 + 4/\theta)}\right)}{P_f \exp\left(\frac{r_1^2}{\sigma^2(2 + 4/\theta)}\right) + (1 - P_f) \exp\left(\frac{r_0^2}{\sigma^2(2 + 4/\theta)}\right)} > t \quad (39)$$

and decide H_0 otherwise. Since it is reasonable to assume that $P_d > P_f$, some algebraic manipulation of (39) leads to the following conclusion: nontrivial decision is reached only when

$\frac{P_d}{P_f} > t > \frac{1-P_d}{1-P_f}$ and therefore, the equivalent likelihood ratio

test is given by the following rule:

Decide H_1 if $r_1^2 - r_0^2 > \eta$, where η is a real number. This establishes that $r_1^2 - r_0^2$ is a sufficient statistic for the given decision problem.

(ii) Conditional Error Probability

By denoting $S_1 = r_1^2$ and $S_0 = r_0^2$, it can be easily established that, under the transmission of frequency f_1 by the sensor, S_1 and S_0 are independently distributed as exponential random variables with means $\lambda_s = (\theta \sigma^2 + 2 \sigma^2)$ and $\lambda_0 = 2 \sigma^2$, respectively. Therefore, the conditional probability of channel error, given that the frequency f_1 was sent, is

$$P_{C1} = P(S_1 - S_0 < \eta) \quad (40)$$

Similarly, the other conditional error probability occurs when frequency f_0 was sent:

$$P_{C0} = P(S_1 - S_0 > \eta) \quad (41)$$

Standard techniques for the evaluation of the probabilities (40) and (41) lead to the final result:

For $\eta \geq 0$

$$P_{C1} = 1 - \frac{\exp(-\eta/\varepsilon)}{1 + 1/\varepsilon}$$

$$P_{C0} = \exp(-\eta/\varepsilon) / (1 + \varepsilon) \quad (42)$$

For $\eta < 0$

$$P_{C1} = \exp(-|\eta|/\varepsilon) / (1 + \varepsilon)$$

$$P_{C0} = 1 - \frac{\exp(-|\eta|/\varepsilon)}{1 + 1/\varepsilon}$$

where $\varepsilon = \left(\frac{\lambda_s + \lambda_0}{\lambda_0} \right)$ is one plus the SNR. Equations

below show the relations between the probabilities of false alarm and detection of a decision, as seen at the fusion center, and the sensor false alarm and detection probabilities:

$$\alpha = P_f(1 - P_{C1}) + (1 - P_f)P_{C0} \quad (43)$$

$$\beta = P_d(1 - P_{C1}) + (1 - P_d)P_{C0} \quad (44)$$

Figs. 5 and 6 show the graphs obtained for BPSK and BFSK modulations. For a given α , BPSK provides a higher detection probability β than FSK, as to be expected. In either modulation, if the matched filter threshold is adjusted to yield a α close to P_f , then considerable loss in detection probability ($\beta \ll P_d$) could occur at low channel SNR values. At SNR of 5 dB, this loss is small for PSK and is only slightly worse for FSK. In the case of a large sensor network, an optimal choice of $\tau(\eta)$ for BPSK (BFSK) can be determined by optimizing the error exponents. Such an optimization procedure requires the knowledge of P_f , P_d and average channel SNR.

IV. CONCLUSION

In this paper we derived asymptotic probability of error exponents for the decentralized detection problem involving a large number of identical sensors and slow Rayleigh fading

links. At the fusion center, the performances of MRC, EGC, SLC, and decision counting rules were analyzed. Also, for a single sensor-to-fusion link, by varying the matched filter threshold, the variation of probability of detection, as a function of probability of false alarm at the fusion center, is evaluated. The results obtained here will be useful in the design of a large wireless sensor network for achieving best detection performance.

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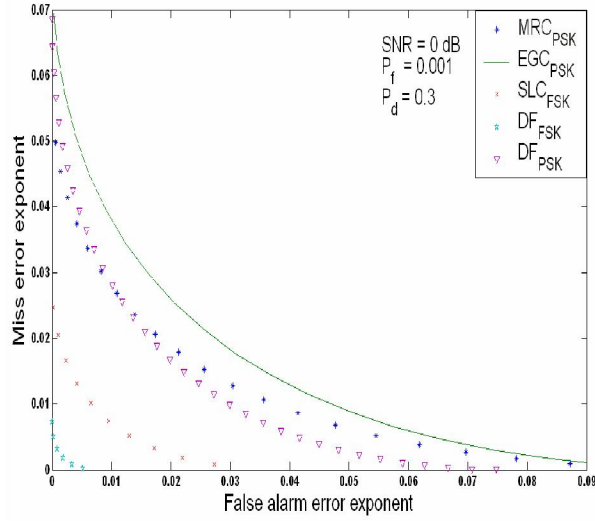


Fig.1 Asymptotic Error Exponents, Miss versus False Alarm

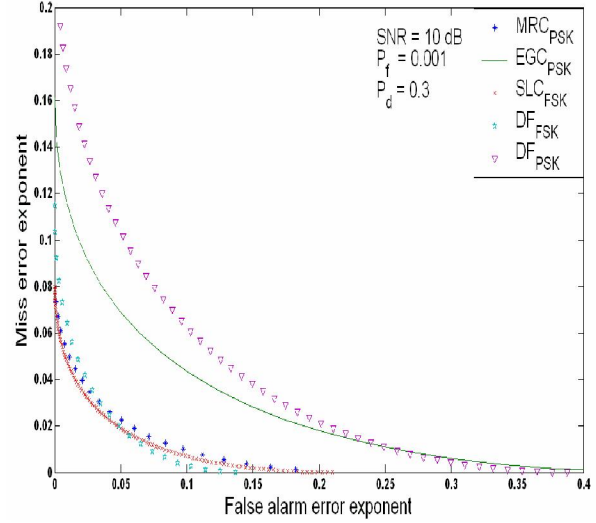


Fig.4 Asymptotic Error Exponents, Miss versus False Alarm

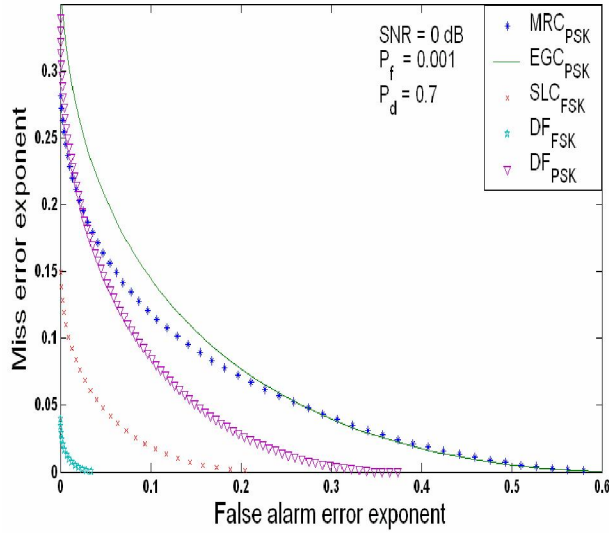


Fig.2 Asymptotic Error Exponents, Miss versus False Alarm

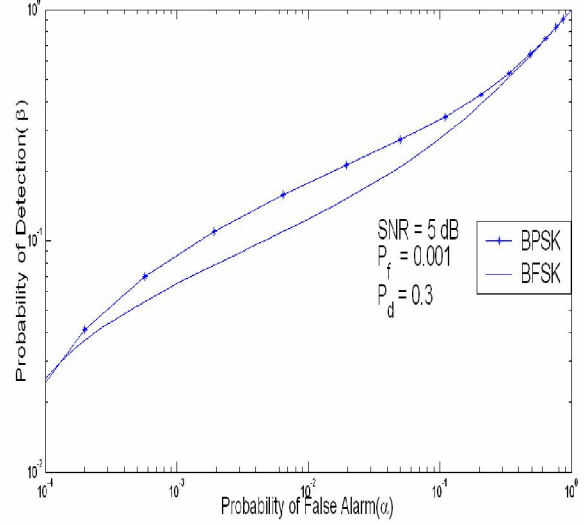


Fig.5 Single sensor, Detection probability Vs. False Alarm probability

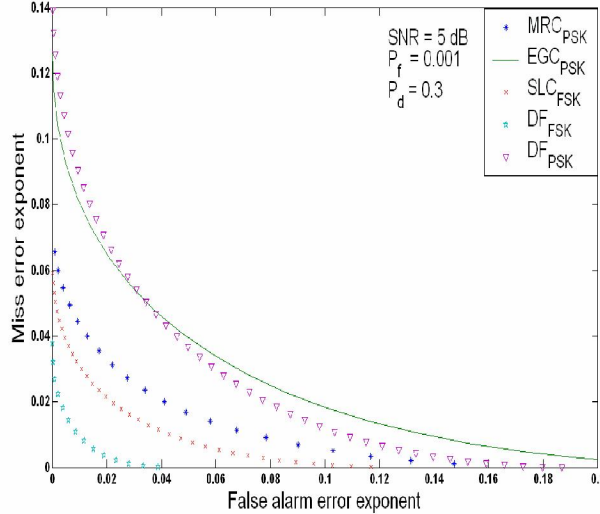


Fig.3 Asymptotic Error Exponents, Miss versus False Alarm

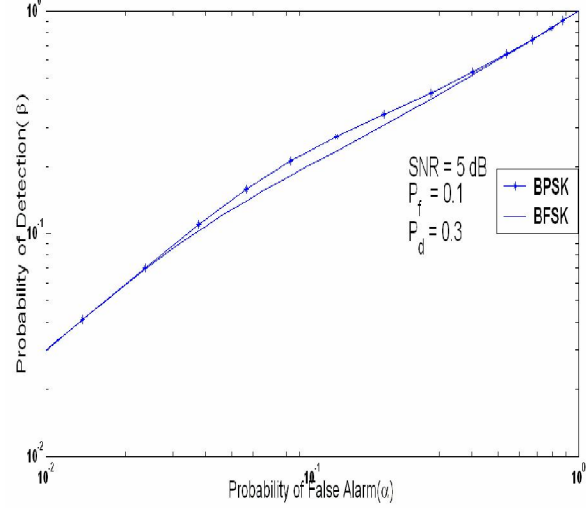


Fig.6 Single sensor, Detection probability Vs. False Alarm probability